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Adaptive Truss Manipulator Space Crane Concept

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An adaptive truss manipulator (ATM) space crane concept for in-space assembly and construction is described in this paper. The underlying mechanism of ATM operation is based on the geometric transformation of its constituent truss modules. In this paper, kinematic description of an ATM is based on the Jacobian matrix derived via the solution technique developed in truss structural analysis, i.e., "extended method of joints." The algorithm based on this method is of computational complexity $\mathcal{O}(m^2)$ where m is the number of truss members in the ATM. Numerical simulations were performed to demonstrate the ATM's key features including deployment/retraction, articulation, and operation with bracing strategy. Such features, especially the bracing strategy, offer a great deal of potential in meeting the operational requirements of the in-space assembly and construction scenarios.

Nomenclature

$[B]$	= direction cosine matrix ($m \times n$)
$[C]$	= $[B]^{-T}$ Jacobian matrix
$[C_{pq}]$	= subpartition of $[C]$ matrix (same for $[C_{po}]$, $[C_{oq}]$, and $[C_{oo}]$)
$[C_{pq}]^+$	= pseudo inverse of $[C_{pq}]$
$\{f\}$	= joint force vector ($n \times 1$)
J	= optimization objective function
m	= number of truss members
n	= number of unconstrained joint degrees of freedom
p	= number of designated trajectory degrees of freedom
$[Q]$	= actuator weighting matrix ($q \times q$)
q	= number of variable length actuators
$\{s\}$	= member force vector ($m \times 1$)
$[T]$	= transformation matrix of truss member (3×3)
$\{u\}_i$	= displacement vector of ith joint in local coordinate
$\{\Delta u\}$	= member length change vector ($m \times 1$)
$\{\Delta u\}_q$	= length change vector of actuators ($q \times 1$)
$\{\xi\}_i$	= displacement vector of ith joint in global coordinates (3×1)
$\{\xi\}$	= displacement vector of all unconstrained degrees of freedom in global coordinates ($n \times 1$)
$\{\xi\}_p$	= displacement vector of designated trajectory degrees of freedom ($p \times 1$)
$\{\xi\}_o$	= displacement vector of remaining degrees of freedom $[(n-p) \times 1)]$

Introduction

THE capability of performing in-space assembly and construction is considered to be a necessary precursor to the development of large spacecraft for future missions such as a Lunar Transfer Vehicle or Mars Transfer Vehicle. It is expected that the mass and volume associated with these mission vehicles will exceed any planned single-launch capacity. For example, a Lunar Transfer Vehicle may weigh up to 200 metric tons and be over 15 m across. As a result, vehicle components or subsystems need to be transferred to a dedicated orbiting construction facility where final assembly and checkout will be completed. Such a construction facility may compose a large scale structural framework as illustrated in Fig. 1.[1] Although such a construction facility will undoubtedly rely extensively on telerobotics and automation, some of the assembled vehicle components and subsystems will probably be too large and too massive for the space telerobotic systems. To overcome the potential difficulty in such construction scenarios, a lightweight truss manipulator space crane concept capable of manipulating and positioning large and massive payloads has been investigated for the past few years.[2,3] The most significant advantage of using a truss manipulator over a conventional mechanical arm is the truss construction's superior stiffness-to-weight ratio. The other advantage is that the truss construction makes it possible for the in-space erection or deployment that offers a potentially tight package for Earth-to-orbit transfers and moderate degrees of mobility in space.

As shown in Fig. 1, the space crane concept proposed in Refs. 2 and 3 is a multijoint, multilink anthropomorphic truss manipulator in which adjacent links made of fixed geometry truss segments are joined together by a single-axis hinge and articulated by the offset linear actuator(s). In this paper, a truss manipulator concept using adaptive structures approach is described for in-space assembly and construction applications. This type of truss manipulator, also known as the variable geometry truss or deployable/controllable geometry truss in literature, was first investigated by Miura et al. in Refs. 4 and 5 in which the basic kinematics of a variable geometry truss manipulator was established. Since then, various aspects of the adaptive truss manipulator have been examined by several researchers, including joint requirements,[6] inverse kinematic analysis,[7,8] motion planning,[9] and laboratory demonstrations.[10,11] An adaptive truss manipulator promises a

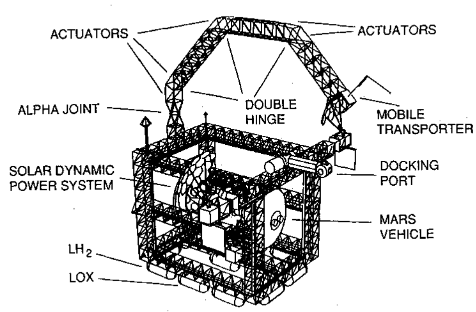

Fig. 1 Concept of an in-space assembly and construction facility.

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demonstrations.[10,11] An adaptive truss manipulator promises a number of advantages over an anthropomorphic one. Among them are its compact stowage volume for in-space storage and mobility, option for deployment as needed, high dexterity in complex workspace, and high redundance of the actuator function. Such features offer a great deal of potential in meeting the operational requirements of an in-space construction facility.

In this paper, a bracing strategy originally proposed by Book et al.[12,14] for ground-based robotic operations is also examined for space crane operations. The bracing strategy requires that the space crane be stabilized by periodically docking with (or connecting to) the construction facility at prelocated bracing posts, such that a stiffer support required for precision assembly operations can be achieved.

In most of the adaptive truss manipulator studies,[4-11] the kinematic description is typically formulated by using the recursive geometric relations of the repeating truss modules. In this paper, kinematic description of an adaptive truss manipulator is based on a non-recursive approach in which the Jacobian matrix is derived via the solution techniques developed in the truss structural analysis, i.e., "extended method of joints."[15,16] One advantage of this approach is the flexibility in dealing with configuration changes as a result of bracing operation (i.e., from open-link to closed-link configuration after bracing).

Adaptive Truss Manipulator Concept

An adaptive truss manipulator (ATM) typically consists of a periodic series of basic truss modules containing members equipped with variable-length actuators. The underlying mechanism of ATM operation is based on the geometric transformation of its constituent truss modules. Thus, by adjusting the variable-length members in the basic module, the ATM configuration, as well as its end effector position and orientation, become predictable and controllable. To perform a stress-free geometric transformation, the entire ATM or the controllable portions of the ATM must be constructed in a statically determinate fashion with the appropriate joints.[6]

In this paper, an ATM made of a series of octahedral modules containing variable-length actuators in all batten members (Fig. 2) is studied. The advantages of an ATM mentioned in the previous section comes at the expense of a large number of variable-length actuators and increased complexity of the manipulator control. To facilitate the control and maintenance of the proposed ATM, the variable-length members may be grouped hierarchically: one with a simple adjustment scheme for deployment and retraction; the other with a more complicated mechanism for articulation and high dexterity operations.

Kinematic Descriptions

An ATM differs fundamentally from a multijoint, multilink anthropomorphic manipulator in its kinematic behavior.

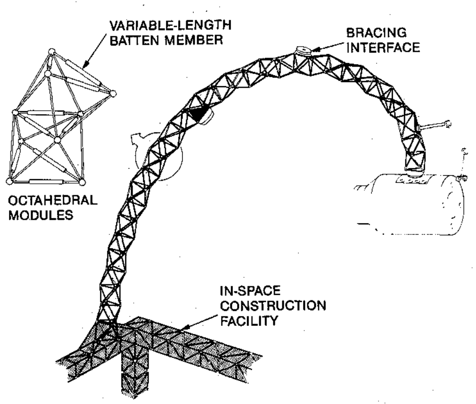

Fig. 2 Adaptive truss manipulator (ATM) concept.

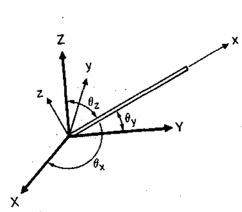

Fig. 3 Coordinate systems of a truss member.

Since the configuration of an ATM is defined by the positions of its joints, it is natural to formulate the kinematic analysis using joint displacements as the independent variables. For each member, the joint displacements can be described in both local and global cartesian coordinates (Fig. 3) which can be related through the coordinate transformation, i.e.,

$$\{u\}_i = [T]\{\xi\}_i \tag{1}$$

where $\{u\}_i = [u_i \; v_i \; w_i]^T$ and $\{\xi\}_i = [U_i \; V_i \; W_i]^T$ are displacements of the ith joint in local and global coordinates, respectively. The 3×3 transformation matrix $[T]$ contains the direction cosines defined by local and global axes. For the truss member shown in Fig. 3, the member length change Δu can be obtained from the relative axial displacement between joints j and i, i.e.,

$$\Delta u = u_j - u_i = [-\cos(\theta_x) \; -\cos(\theta_y) - \cos(\theta_z) \; \cos(\theta_x) \; \cos(\theta_y) \; \cos(\theta_z)] \begin{Bmatrix} U_i \\ V_i \\ W_i \\ U_j \\ V_j \\ W_j \end{Bmatrix} \tag{2}$$

where $\cos(\theta_x)$, $\cos(\theta_y)$, and $\cos(\theta_z)$ are direction cosines of the member axial axis defined in the global coordinate system. From Eq. (2), a system of geometric compatibility equation can be assembled according to the joint displacement degrees of freedom, i.e.,

$$\{\Delta u\} = [B]^T\{\xi\} \tag{3}$$

where the m-element vector $\{\Delta u\}$ contains the members' length changes, and the n-element vector $\{\xi\}$ contains the resulting joint displacements in global Cartesian coordinates Each column of the $m \times n$ matrix $[B]^T$ contains each member's direction cosines as described in Eq. (2) augmented with zeros to the full order. For a statically determinate truss, the number of truss members must be equal to the number of the unconstrained joint degrees of freedom, i.e., $m = n$. This leads to a square and nonsingular $[B]$ (or $[B]^T$) matrix such that $[B]^{-T} = [C]$ exists and

$$\{\xi\} = [C]\{\Delta u\} \tag{4}$$

Equation (4) describes the direct kinematic behavior of an ATM. In other words, given the length changes of the variable-length actuators, the resulting configuration change can be obtained from the Jacobian matrix $[C]$. As a result, a continuous ATM motion can then be simulated by an incremental analysis using Eq. (4).

Clearly, the effectiveness of the kinematic description of Eq. (4) depends on the computational complexity of obtaining $[B]^{-1}$ from the highly sparse $[B]$ matrix. From elementary structural analysis,[17] the relationship between member forces and joint forces is described by the joint equilibrium equation, i.e., $[B]\{s\} = \{f\}$. Using this relationship, one can construct the $[B]^{-1}$ matrix, column by column, from the resulting member forces induced by a unit force at one joint degree of freedom at a time, i.e., $[B]^{-1} = (\{s\}_1\{s\}_2 \cdots \{s\}_n)$. To solve for the member forces, a fast and storage efficient algorithm based on the extended method of joints[18] (i. e., the method of joints combined with the generalized Henneberg method) was utilized in this study. As a result, this algorithm is of computational complexity $\mathcal{O}(m^2)$ and requires only $\mathcal{O}(m)$ primary storage, as compared with the computational complexity of $\mathcal{O}(m^3)$ of direct matrix inversion.

One of the purposes of an ATM is to transport a payload to its destination along a desired trajectory; this motion depends on the combined actions of the variable-length actuators. The task of determining the corresponding length changes in the actuators for a given payload trajectory is an inverse kinematics problem. Even though an ATM is featured by the high redundancy in the actuator degrees of freedom, it is not necessary that every truss member be equipped with the variable-length actuator. Thus, Eq. (4) can be partitioned in such a way that

$$\begin{Bmatrix} \xi_p \\ \xi_o \end{Bmatrix} = \begin{bmatrix} C_{pq} & C_{po} \\ C_{oq} & C_{oo} \end{bmatrix} \begin{Bmatrix} \Delta u_q \\ 0 \end{Bmatrix} \tag{5}$$

where $\{\xi_p\}$ are p degrees of freedom associated with the given trajectory, $\{\xi_o\}$ are the remaining unspecified degrees of freedom, $\{\Delta u_q\}$ are the length changes of q actuators, and $[C_{pq}]$, $[C_{po}]$, $[C_{oq}]$, and $[C_{oo}]$ are the submatrices partitioned from the $[C]$ matrix of Eq. (4), accordingly. The upper part of Eq. (5) describes the relationship between the given trajectory and required actuator actions, i.e.,

$$\{\xi_p\} = [C_{pq}]\{\Delta u_q\} \tag{6}$$

For an ATM, it is expected that the number of members equipped with the variable-length actuator is much greater than the number of degrees of freedom associated with the given trajectory, i.e., $q > p$. When the rank of $[C_{pq}]$ is less than p, Eq. (6) represents a singular problem. In other words, the ATM is not capable of displacing itself according to the given trajectory. When the rank of $[C_{pq}]$ is p and $p < q$, there is no unique solution $\{\Delta u_q\}$ to Eq. (6). Thus, additional degrees of freedom in the choice of $\{\Delta u_q\}$ can be used to satisfy certain optimality criterion. In this study, minimization of the weighted vector norm of the member length changes is employed, i.e.,

$$\text{Minimizing } J = \{\Delta u_q\}^T[Q]\{\Delta u_q\} \tag{7}$$

subject to the constraint of Eq. (6).

The $[Q]$ matrix contains the weighting coefficients of the actuator actions. The solution to this optimization problem can be found in the literature such as Ref. 19, i.e.,

$$\{\Delta u_q\} = [C_{pq}]^+\{\xi_p\} \tag{8}$$

where $[C_{pq}]^+ = [Q]^{-1}[C_{pq}]^T([C_{pq}][Q]^{-1}[C_{pq}]^T)^{-1}$ is commonly referred to as the pseudo or generalized inverse of the $[C_{pq}]$ matrix. From Eq. (8), given a trajectory, the required actuator actions can be determined for the optimality condition of Eq. (7).

Basic ATM Operations

Based on the kinematic analysis described in the previous section, some of the basic operations of an ATM are examined in this section through numerical simulations. To gain insight into the ATM operations without being burdened by the complicated geometric transformation in a three-dimensional space, a two-dimensional representation of the octahedral truss, Fig. 4, is employed in the following study. The two-dimensional ATM model consists of 30 bays containing variable-length batten members and 4.03-m fixed length lateral members. In stowage, Fig. 5, the ATM is 15 m in height with the batten in its maximum length of 8 m. The member properties listed in Table 1 are selected to be the same as those of Ref. 3. The basic operations discussed in this section include deployment/retraction, articulation, dexterous motions, and bracing strategy.

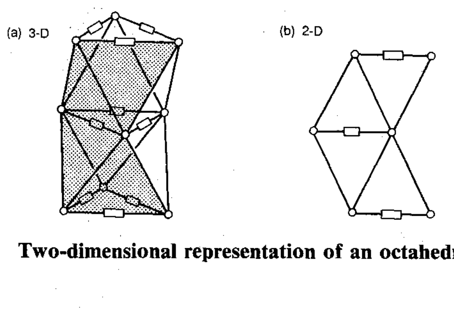

Fig. 4 Two-dimensional representation of an octahedral truss.

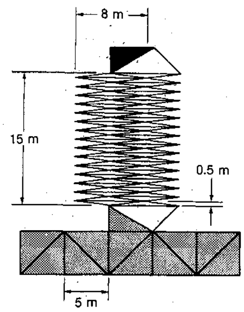

Fig. 5 Two-dimensional ATM in stowage.

Table 1 ATM truss member properties

Area	3.006 cm^2
Modulus	215.5 GPa
Density	1882 kg/m^3

Deployment/Retraction

Two key features of an ATM concept are that it can be stowed in a compact volume for storage and in-space mobility, and deployed only when it is needed. Such a deploy-as-needed option together with the high mobility offers a great deal of flexibility in meeting the operational requirements of an in-space construction facility. For example, an ATM can be located in such a position that its deployment/retraction motions and payload fetching operation from an orbiter will cause least momentum distrubances to the facility's attitude controller.

There are two approaches for deploying or retracting an ATM: a simultaneous mode and a sequential mode.[4] In the simultaneous mode, the variable-length actuators are controlled simultaneously, while in the sequential mode the actuators are controlled sequentially on a module by module basis. In either case, the deployment or retraction operation can be treated as an inverse kinematics problem in which a straight or curved trajectory can be specified. As an example, a simultaneous deployment sequence of an ATM from stowage to the extended 100-m configuration is illustrated in Fig. 6. After deployment, the batten length and bay height are changed from 8 to 4.7 m and 0.5 to 3.3 m, respectively. Without any payload, the natural frequencies of the extended ATM of Fig. 6 are 0. 44, 2.73, and 7.51 Hz for the first, second, and third modes, respectively. The other feature of an ATM is that it can also be partially deployed, Fig. 7, based on the in-situ requirements of the construction and assembly scenarios.

Articulation

Although an ATM is typically equipped with a highly redundant number of actuators, the basic articulation function of a conventional multijoint, multilink truss manipulator can be accomplished by an ATM using only a fraction of the available actuators. To facilitate an ATM's articulation control, it is desirable to configure groups of actuators as the

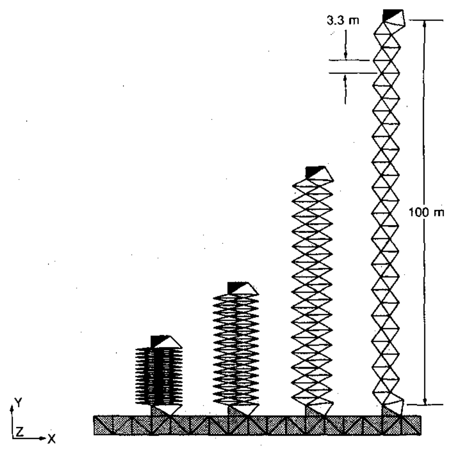

Fig. 6 Two-dimensional ATM deployment sequence.

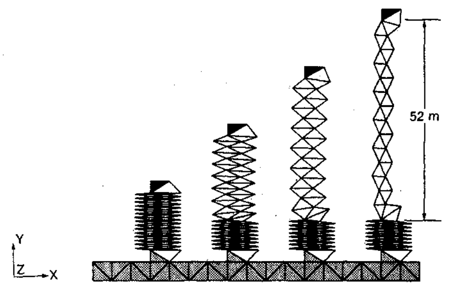

Fig. 7 Two-dimensional ATM partial deployment sequence.

articulation modules of the ATM. For example, an articulation module shown in Fig. 8a consists of three bays of the two-dimensional ATM. Through geometric transformation, the articulating module can be effectively rotated by adjusting the batten length in either direction. Direct kinematic analysis results of a 1-m length change in batten members whose nominal length is 5 m are shown in Figs. 8b and 8c. In Fig. 8b, a 29.7 deg rotation is introduced by adjusting only every other batten member in the same direction; while a 56.4 deg rotation is attained by adjusting every batten member in alternatively reversed directions, i.e., 1-m length change in extension and contraction, respectively. In the following example, four articulating modules are defined for the 30-bay ATM model as shown in Fig. 9. The reach envelopes resulting from a 1-m length change in articulating modules are also illustrated in Fig. 9 for the ATMs with one, two, three, or four articulating modules.

Bracing Strategy

Among all of the advantageous features offered by the ATM concept, the proposed bracing strategy is believed to be the most valuable to the in-space assembly scenarios. Like most of the robotic operations, the space crane has a two-tier operation strategy, i.e., gross- and fine-motion phases. The gross motion of a space crane involves mainly a distant transportation of payloads, while the fine-motion phase involves the precision assembly operations by the space crane-borne mechanical manipulator(s). In the fine-motion phase, the space crane serves as a working platform for the mechanical manipulator(s). The lightweight and long reach requirements together with the potentially massive payloads, however, can result in the extremely low natural frequencies of the space crane. The presence of low natural frequencies makes the space crane turned working platform susceptible to the disturbances arising from other assembly/construction activities within the facility or payload maneuvering. To overcome such a difficulty, an operational strategy which involves "bracing" the space crane against the construction facility is proposed to effectively provide a stiffer working platform for the fine-motion phase. To implement the bracing strategy, bracing interfaces requiring crude alignment and simple docking mechanism can be prelocated throughout the construction facility.

Clearly, conventional multijoint, multilink truss manipulators, such as the one shown in Fig. 1, are not suitable for the bracing operations in a complex work space. On the other hand, the ATM's inherent dexterity makes it an ideal manipulator to implement bracing strategy. The inherent dexterity of an ATM is attributed to the adaptive structure concept in which a highly redundant number of actuators are available. To demonstrate the extreme of ATM dexterity, the resulting ATM configurations are shown in Fig. 10a when all the actuators were commanded to extend 0.5 m, and Fig. 10b when commanded to contract 1.0 m from the initial configuration of Fig. 6.

The benefits of bracing strategy in ATM operation is demonstrated in the example depicted in Fig. 11, in which a 5000 kg payload was to be positioned with high precision near the end of a construction facility. Without explicitly comparing the achievable positioning accuracy, an implicit figure of merit, i.e., the ATM's natural frequencies, was calculated for ATMs with and without bracing strategy. In both cases, the payload was transported from the initial configuration of Fig. 6 to the final configuration of Fig. 11a or 11b. Without bracing strategy, it is shown that the final destination can be achieved by activating only those actuators in the articulating modules as defined in Fig. 9. In this case, the lowest natural frequency of the final ATM configuration (Fig. 11a) is 0.07 Hz. To incorporate bracing strategy, the ATM was first commanded to dock with the bracing point by following the trajec-

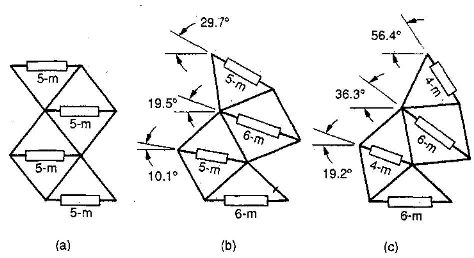

Fig. 8 Rotation of an articulating module.

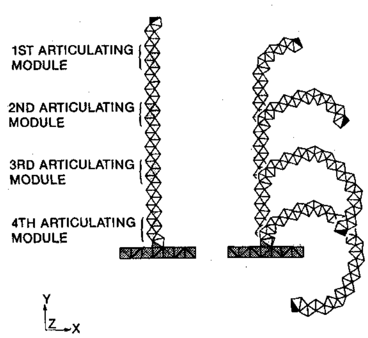

Fig. 9 Reach envelopes of a two-dimensional ATM.

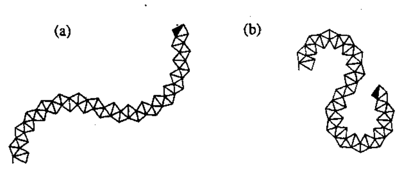

Fig. 10 ATM in high dexterity.

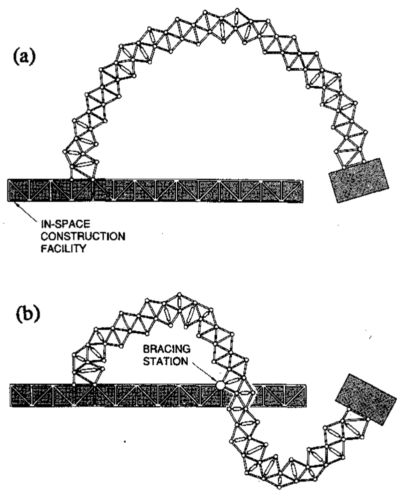

Fig. 11 ATM bracing strategy.

tories defined by the bracing point. Again, it was shown that this phase of operation can be achieved by the actuators in the articulating modules alone. After bracing, the payload was subsequently transported to the final destination of Fig. 11b by activating all actuators in between the payload and bracing point. Notice that after the bracing operation, the ATM configuration was altered from a simple open link manipulator to the one having both open and closed links. The lowest natural frequency of the final ATM configuration of Fig. 11b is increased approximately by a factor of 3 to 0.22 Hz. In other words, the braced ATM is now approximately one order of magnitude stiffer than an unbraced one.

Conclusions

An adaptive truss manipulator concept was described for the in-space assembly and construction facility. The underlying manipulator mechanism—geometric transformation through member length changes—was illustrated. An adaptive truss manipulator differs fundamentally from a conventional multijoint, multilink anthropomorphic manipulator in its kinematic behavior. In this paper, the kinematic description was formulated based on a nonrecursive approach, i.e., a system of geometric compatibility equations of the complete manipulator. The algorithm was shown to have computational complexity $\mathcal{O}(m^2)$, which is encouraging for the development of a real-time control of an adaptive truss manipulator. Advantages of an adaptive truss manipulator over a conventional truss manipulator were described and illustrated through numerical simulations. Among them are its compact stowage volume for in-space storage and mobility, option for deployment as needed, and high dexterity for bracing strategy to improve positioning accuracy. The basic operations of an adaptive truss manipulator including deployment retraction, articulation, and bracing strategy were demonstrated numerically. With a single bracing point, an ATM can be one order of magnitude stiffer than the unbraced one. With the multipoint bracing, it is expected that benefit to the positioning accuracy and precision assembly will be dramatically improved. Furthermore, when assembling multiple subsystems together, the adaptive truss manipulator can be mounted on one of them, thereby eliminating disturbances to the construction facility and facilitating routine control of the facility. With these advantageous features, it is believed that an adaptive truss manipulator can offer a great deal of potential in meeting the operational requirements of the in-space assembly and construction scenarios.

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